

$(P, s_{MP})$  and  $r_{\nu, s}(P, s_{MP}) = 1 - a_{\nu, s}(P, s_{MP})$ , spectral directional absorption coefficients and reflection coefficients of the surface at point P for radiation at frequency  $\nu$  coming from point M;  $K(P, M) = (\cos \theta_P \cos \theta_M) / \pi r_{MP}^2$ , kernel of the integral equation of heat transfer, known from theory and a function of points M, P in the given system;  $a_{\nu}(P) = \frac{1}{\pi} \int_{(2\pi)} a_{\nu, s}(P, s) \cos(s, n_P) d\omega_s$ ,  $r_{\nu}(P) = 1 - a_{\nu}(P)$ , respectively, spectral hemispherical absorption coefficient and reflection coefficient of the surface at point P for ideal diffuse radiation at frequency  $\nu$ ;  $E_{r, k}^{(giv)}$ ,  $T_{w, k}^{(giv)}$  and  $E_{T, k}^{(giv)}$ , quantities as stipulated;  $E_{in, i}^{(n)}$  and  $E_{in, i}^{(n+1)}$ ,  $T_{w, i}^{(n)}$  and  $T_{w, i}^{(n+1)}$ , quantities defined in n-th and (n + 1)-th approximations of the iteration process;  $d\omega_{s'}$ , element of the solid angle about direction  $s'$ ;  $s'$ , unit vector in the given direction;  $I_{i-n}(s)$ , integral intensity of effective radiation within the i-th band in direction  $s$ ;  $n$ , unit vector normal to the boundary surface at a given point;  $\nu_{min, i}$  and  $\nu_{max, i}$ , respectively, lower limit and the upper limit of the i-th band;  $\rho_{\nu}^{(V)}(T; s', s)$  and  $\rho_{\nu}^{(F)}(T_w, s', s)$ , spectral coefficients of brightness of the medium and for the surface, respectively;  $k_{\nu}^{(T, \nu)}$  and  $\alpha_{\nu}^{(T, \nu)}$ , respectively, spectral attenuation coefficient and spectral absorption coefficient for the given medium;  $n_{\nu}$ , spectral refractive index for that medium; and  $I_{o, \nu}(\nu, T)$ , spectral intensity of equilibrium radiation in vacuum.

#### LITERATURE CITED

1. V. N. Adrianov, "Accuracy of 'gray' approximation," *Teplofiz. Vys. Temp.*, 19, No. 5, 1014-1017 (1981).

#### TEMPERATURE DISTRIBUTION IN THIN FILMS RECEIVING RADIANT ENERGY

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An expression is derived for estimating the temperature distribution in thin metal and semiconductor films locally receiving radiant energy.

The operating modes and the design configurations of thin-film devices are known to depend on the number and the characteristics of influencing external factors (electric current, magnetic field, temperature, etc.). Thermal action is of special significance among such factors, inasmuch as, even when none is assumed to take place, temperature fields can build up as a result of conversion of other forms of energy to heat.

It has been estimated in earlier studies [1-3] how the thermal (superposing) galvanothermodynamic effects in semiconductor bulk specimens influence the accuracy of determination of the underlying galvanothermodynamic effects, and the feasibility of determining several thermophysical parameters of specimens on the basis of complete separation of all measurable effects has been established. As far as these authors know, no such study was ever made with regard to film specimens.

Other studies [4-6] have dealt with the temperature distribution and the thermal fluxes in thin films resulting from action of laser radiation. It has been established that, as a rule, the thermal flux  $\Phi_2$  to the substrate exceeds the thermal flux  $\Phi_1$  in the plane of the film (except in two cases: during a short initial time period of the order of  $10^{-8}$  sec, and when the substrate has a low thermal conductivity).

In some problems, such as recording the distribution of energy flux density in the focal spot of solar concentrators [7, 8], or calculating the thermal distortion of the contour of the laser irradiation zone [4-6, 9], or determining the buildup time of superposing galvano-

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thermomagnetic effects in thin films, it is, however, the thermal flux  $\Phi_1$  which plays the more significant role. For recording the radiation from high-temperature sources with long exposure time, moreover, one uses ferromagnetic films [7, 8] operating at temperatures near the Curie point of the materials.

In one study [9], the temperature field has been calculated in a metal film adiabatically insulated from the substrate and illuminated with a light beam of rectangular cross section. The expressions obtained there are rather unwieldy, however, inasmuch as they contain double series and thus require a computer for numerical evaluations and plotting of graphs.

Here a simple scheme will be described for solving the problem of heat propagation over the plane of a film, and an approximate relation will be derived in analytical form convenient for estimating and determining temperatures and thermal fluxes.

We consider an infinitesimal segment of the boundary between the heated (illuminated) region and the cold (dark) region of a thin film. Let a thermal flux  $d\Phi_1$  propagate through this segment and then along an infinitesimally narrow semibounded strip. The problem is to determine the temperature at points of the strip at the instant of time  $\tau$  when action of a radiation pulse ends. The solution to a similar problem is given elsewhere [10]; for a uniform initial temperature distribution it is

$$T(x, \tau) = \int_0^\tau 2a \frac{\partial G(x, 0, \tau - t)}{\partial \xi} F(t) dt, \quad (1)$$

with  $G(x, \xi, t)$  denoting the source function and  $F(t)$  denoting the temperature at the hot end of the strip (boundary condition of the first kind). The form of function  $F(t)$  can be determined from the exact solution to the problem of heat conduction, but we will replace it with its maximum value  $T_B$  (thus possibly somewhat overestimating  $T(x, \tau)$ ). Then, after  $T_B$  and

$$\frac{\partial G(x, \xi = 0, t - \tau)}{\partial \xi} = x(16\pi)^{-1/2} [a(t - \tau)]^{-3/2} \exp(-x^2/4a(t - \tau))$$

have been inserted into expression (1), integrating the latter yields the relation

$$T(x, \tau) \leq T_B [1 - \operatorname{erf}(x/(4a\tau)^{1/2})] \quad (2)$$

for estimating the temperature at point  $x$  at time  $\tau$ . Here  $x$  is the distance from the boundary of the heated region to the given point in the cold region. This relation can be expressed in the more convenient form

$$\operatorname{erf}(x/(4a\tau)^{1/2}) \leq 1 - \frac{T(x, \tau)}{T_B}. \quad (3)$$

When the temperatures  $T(x)$  of interest do not differ from  $T_B$  by more than a few percent, then the error function can be linearized to ( $\operatorname{erf}(y) \approx 2(\pi)^{-1/2}y$ ). In this case the relation becomes

$$x \leq (\pi a \tau)^{1/2} \frac{T_B - T(x)}{T_B}. \quad (4)$$

Thus, expression (2) yields an estimate of the film temperature at a stipulated distance  $x$  from the boundary of the light spot, while expressions (3) and (4) yield the distance from that boundary to the isotherm of a stipulated temperature. Parameters in these relations are the thermal diffusivity  $a$  of the film material, the exposure time  $\tau$ , and the boundary temperature  $T_B$ .

In another study [4] an expression similar to ours is given:  $\Delta r \approx 0.5(\pi a \tau)^{1/2} \Delta q/q_{1S}$ . There  $\Delta r$  is the deviation of the dimension of the evaporation region from the initial dimension of the light beam and  $\Delta q$  is the deviation of the actual beam power from the theoretical one. It is easy to see that this relation remains valid only for  $\Delta q/q_{1S} \ll 1$ .

Expressions (2)-(4) yield higher-than-real temperatures  $T$  and distances  $x$ , because of at least three assumptions made here: 1) that the insulation between film and substrate is adiabatic; 2) that the boundary of the lighted region is rectilinear (at a curvilinear convex

TABLE 1

Isotherms (dimensionless)	0,5 and 0,4	0,5 and 0,3	0,5 and 0,2	0,5 and 0,1	0,5 and 0,6
$x, \mu\text{m}$ :					
acc. to [9]	0,25	0,50	0,80	1,22	0,18
acc. to (3)	0,19	0,40	0,64	0,86	0,16

boundary segment the heat actually propagates not along a strip but along a sector, i.e., a figure with large area); 3) that the boundary temperature  $T(t)$  is equal to its maximum  $T_B$ .

Let us apply these relations to a few examples.

Estimating the Error of Thermomagnetic Recording of Optical Images. In recording of optical images on thin ferromagnetic film, as well as in technological treatment of films with laser radiation, there is one intermediate stage required, namely formation of a thermal image, i.e., a certain temperature distribution [7, 11]. During this stage, owing to relaxation of elements of the thermal image, distortions appear in the recorded optical image or produced film configuration.

As has already been mentioned [11], during readout of an image recorded on thin Permalloy film ( $\alpha_{\text{perm}} = 12 \text{ mm}^2 \cdot \text{sec}^{-1}$ ) it is impossible to distinguish segments with a temperature difference  $\Delta T \leq 1-3^\circ\text{C}$ . The distance  $x$  between such indistinguishable isotherms constitutes a nonremovable absolute recording error due to diffusion of heat. This error can be estimated according to relation (3). Let  $\Delta T = 5^\circ\text{C}$ ,  $T_B = 200^\circ\text{C}$  (the Curie point for Permalloy is  $\sim 700^\circ\text{C}$ ), and  $\tau = \{10^{-6}, 10^{-3}, 1\}$  sec. Then expression (3) yields  $\Delta x \leq 0.2, 6.3,$  and  $200 \mu\text{m}$ , correspondingly.

Estimating the Magnitude of Thermal Flux. As long as the inequality  $\Delta T \ll T_B$  or the equivalent inequality  $x \ll (\alpha\tau)^{1/2}$  holds true, the ratio of small finite quantities  $\Delta T$  and  $x$  ( $\approx \Delta x$ ) will, with acceptable accuracy, represent the derivative  $\partial T/\partial x$  so that relation (4) can be rewritten as

$$\frac{T_B}{(\pi\alpha\tau)^{1/2}} \lesssim \frac{\Delta T}{\Delta x} \approx \frac{\partial T}{\partial x} = \frac{|\Phi_1|}{\lambda}, \text{ i.e., } |\Phi_1| \gtrsim \frac{\lambda T_B}{(\pi\alpha\tau)^{1/2}}. \quad (5)$$

Expression (5) yields the magnitude of the specific thermal flux  $\Phi_1$  propagating in the plane of the film away from the boundary of the lighted region. When, e.g.,  $T_B = 200^\circ\text{C}$ ,  $\alpha_{\text{perm}} = 12 \text{ mm}^2 \cdot \text{sec}^{-1}$ ,  $\lambda_{\text{perm}} = 16 \text{ W}/(\text{m} \cdot ^\circ\text{C})$ ,  $p = 1 \text{ cm}$ , and  $d = 1 \mu\text{m}$ , we find that the total thermal flux  $\phi_1$  from the heated region is  $0.53 \text{ W}$  at time  $\tau = 10^{-6}$  sec and a thousand times smaller at time  $\tau = 1$  sec.

Comparison of Theoretical Data [9] and Data according to Relation (3). As was mentioned before, in study [9] graphs (maps) had been plotted of isotherms in a chromium film at the end of a heating pulse of  $\tau = 10^{-8}$  sec duration. Since our assumptions 1) and 2) were also made in those calculations, one would expect the results to be close to those according to relation (3). Indeed, as will be shown subsequently, the difference (with  $\alpha = 29 \text{ mm}^2 \cdot \text{sec}^{-1}$  [12] and  $\tau = 10^{-8}$  sec in expression (3)) does not exceed 20% (such a close agreement indicates that assumption 3),  $T(t) \rightarrow T_B$ , is not essential).

Therefore, the proposed simple expression (3) is quite adequate for estimating temperature fields and thermal fluxes which propagate in a thin film as a result of the latter being heated by sources of radiant energy.

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It is to be noted that expression (3) ranks intermediate between rough approximate estimates of the  $l \sim (\alpha\tau)^{1/2}$  kind for the length of the propagation path of a heat front and exact ones needed in various problems, such as recording semitone images, analysis of processes in films during phase transitions, and film biothermometry.

#### NOTATION

$\Phi_1$  is the thermal flux along the film;  $\Phi_2$ , thermal flux across the film to the substrate;  $T_B$ , maximum temperature at the lighting boundary;  $\tau$ , duration of a radiation pulse;  $\alpha$ , ther-

mal diffusivity of the film;  $\lambda$ , thermal conductivity of the film;  $x$  ( $\equiv \Delta x$ ), distance from the lighting boundary;  $T(x, \tau)$ , temperature at a given point;  $d$ , film thickness; and  $p$ , perimeter of the lighted region.

#### LITERATURE CITED

1. I. S. Lisker and M. B. Pevzner, "Feasibility of complete separation of galvanomagnetic and thermomagnetic effects in semiconductors," *Inzh. Fiz. Zh.*, 30, No. 1, 130-135 (1976).
2. I. S. Lisker and M. B. Pevzner, "Feasibility of determining thermophysical properties of semiconductors from Ettinghausen-effect measurement by method of variation of magnetic field," *Inzh. Fiz. Zh.*, 36, No. 3, 460-465 (1979).
3. I. S. Lisker and M. B. Pevzner, "Determination of thermophysical properties of semiconductors from measurement of superposing galvanomagnetic and thermomagnetic effects by method of variation of influencing factors," *Inzh. Fiz. Zh.*, 37, No. 4, 662-667 (1979).
4. M. N. Libenson and M. N. Nikitin, "Thermal distortions of drawing during laser treatment by projection method," *Fiz. Khim. Obrab. Mater.*, No. 5, 9-13 (1970).
5. N. N. Rykalin, A. A. Uglov, and N. I. Makarov, "Calculation of heating of films by laser radiation," *Fiz. Khim. Obrab. Mater.*, No. 2, 3-8 (1971).
6. G. L. Gurevich and V. A. Murav'ev, "Action of laser radiation on thin films," *Fiz. Khim. Obrab. Mater.*, No. 1, 3-8 (1973).
7. R. B. Bairamov, M. A. Gurbanyazov, N. R. Korpeev, I. S. Lisker, and S. Ya. Yazliev, "Method of determining energy characteristics of radiant energy concentrators," USSR Patent disclosure No. 697781, *Byull. Izobret.*, No. 42 (1979).
8. M. A. Gurbanyazov, I. S. Lisker, and S. Ya. Yazliev, "Experimental module for magneto-optical study of ferromagnetic films," *Izv. Akad. Nauk Tadzh. SSR, Ser. Fiz. Tekh. Khim. Geol. Nauk*, No. 5, 94-97 (1979).
9. V. P. Veiko, E. A. Krutenkova, and G. A. Kotov, "Calculation of thermal distortions of drawing during laser treatment of thin films," *Fiz. Khim. Obrab. Mater.*, No. 5, 37-43 (1980).
10. A. N. Tikhonov and A. A. Samarskii, *Equations of Mathematical Physics* [in Russian], Nauka, Moscow (1966).
11. L. M. Klyukin, B. M. Stepanov, V. A. Fabrikov, and A. V. Khromov, *Photographing on Magnetic Films* [in Russian], Atomizdat, Moscow (1971).
12. G. W. Kaye and T. H. Laby, *Tables of Physical and Chemical Constants*, Longman (1973).